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Construction of learning strategies to combine culture elements and technology in teaching group theory

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ABSTRACT: The new curriculum of higher education is based on the competence of graduates. The competencies that students should develop are comprehending the knowledge and using it to solve problems by explorative and using technology. One of the subjects in mathematics that students learn is group theory. A group is an algebraic structure consisting of a set which fulfils some properties. As written in the new curriculum, learning should instil values, including cultural values, so that students are more appreciative and familiar with the culture of their nation. So, this study aimed to construct learning strategy on teaching group theory with combining culture element and technology. The results of this study were the steps of learning strategies that had been constructed to learn group theory by involving culture and technology that got a positive response from students. The technology referred to in this study was the use of programs, such as Kali, Tess, Group Explorer and jBatik software. Meanwhile, the cultural elements used in group theory learning were the motifs of batik and carving.

INTRODUCTION

Efforts to improve the quality of education have been carried out continuously and thoroughly in Indonesia including the aspects of knowledge, skills, attitudes and values. An effort to improve quality has been effected by a change in the new curriculum; namely, the KKNI-based curriculum. One requirement that must be fulfilled by an undergraduate programme based on KKNI is that the graduates are able to apply their expertise and make use of science, technology, and/or art in their field in problem solving and able to adapt to the situation at hand. Therefore, learning by utilising technology is necessary.

This method has also been implemented by the authors in fractal geometry learning that leads to significant improvements in student competence [1]. With experiment-based activities, the students apply the theories learned in solving problems, such as by applying the fractal dimension of the eye fundus image to classify diabetic retinopathy [2]. One of the expected learning achievements of the new curriculum in addition to knowledge and use of technology, is the cultivation of values, one of which is respecting cultural values.

The importance of learning abstract algebra, especially group theory, is widely acknowledged. Gallian says that *...abstract algebra is important in the education of a mathematically trained person. The terminology and methodology of algebra are used ever more widely in computer science, physics, chemistry, and data communications, and of course, algebra still has a central role in advanced mathematics itself* [3].

Many applications based on group theory have been produced. Yanxi Liu and other researchers explained the applications of group theory in robotics, computer vision, computer graphics and medical image analysis. The understanding of symmetry played a profound role in several important discoveries, including relativity theory, human DNA structure or the quasicrystals [4]. Funk and Liu have created the first deep-learning neural network for reflection and rotation symmetry detection (Sym-NET) that provided an affirmative response to the debate on whether the human perception of symmetry in the wild can be computationally modelled [5].

Group theory is a theory that concerns the characteristics and properties of a group. A group is an algebraic structure consisting of a set which fulfils some properties. Because of its abstract nature, it makes it difficult for students to learn. To make it easier for students to learn, a certain strategy in teaching this subject matter was needed. Some experts used several methods to teach group theory. Cornock suggested teaching group theory using Rubik's cubes and students gave positive responses to the teaching methods and the use of the cubes [6].

Gordon proposed the teaching of group theory using a geometry approach and indicated commercial software packages that can be used to enhance students' understanding [7].

Carter proposed some approaches to group theory learning that were different from traditional approaches, which provide group understanding through activities and provide a visual understanding of the group [8].

Hazzan described ways in which students deal with abstract algebra (group theory) concepts by making these concepts mentally accessible. More specifically, the ways in which students conceive abstract algebra concepts are analysed through the theme of reducing the level of abstraction. As it turns out, in many cases, reducing the level of abstraction is an effective strategy [9].

Over the last two decades, software has become an accepted tool in group theory. Many software products have been created to make it easier for students to learn, explore the properties of the group, including Group Explorer, GAP, Magma, Cayley, Kali and Tess. GAP (groups, algorithms and programming) is a computer algebra system for computational discrete algebra with particular emphasis on computational group theory. Group Explorer is mathematical visualisation software for the abstract algebra that can help the user visualise group theory and builds students' intuition. Magma is a large, well-supported software package designed for computations in algebra, number theory, algebraic geometry and algebraic combinatorics. It provides a mathematically rigorous environment for defining and working with structures, such as groups, rings, fields, modules, algebras, schemes, curves, graphs, designs, codes, and many others. Cayley is a high-level programming language, which has been developed to allow easy access to a collection of algorithms for doing calculations in groups and related structures.

Meanwhile, Kali and Tess is a program to create a pattern visualisation of a group, such as a cyclic group, dihedral group, frieze group or wallpaper group. Hulpke in his paper, said that it often seems hard to predict, which calculations will be easy in practice, and which ones are not feasible at all [10]. Complexity analysis can be helpful, but does not always give a full picture. He gave the survey result about what methods are generally available, which ones only in theory, and what obstacles, sometimes involving algorithmic problems in other areas of mathematics, lie in the way of attacking problems that are currently infeasible [10]. In this study, the strategies chosen to teach group theory by incorporating cultural and technological elements were preceded by the provision of activities organised, so that students understood the concept of the group and its nature by conducting the activity. Then, by using the software, students were asked to explore the properties of the group and by using the symmetry group of batik or carving motif objects, they were expected to understand the symmetry group through the object of local culture. Furthermore, the students were asked to create some motif of batik or carving that would be suitable for each symmetry group type.

LEARNING STRATEGIES TO TEACH GROUP THEORY

A group is an algebraic structure form by a set and a binary operation that satisfies some properties. The definition of a group given by Herstein [11] and Gallian [3], refers to a non-empty set G with a binary operation * which is said to be a group, if the following properties are satisfied:

i. $a*b \in G, \forall a,b \in G$.
ii. There is e in G such that $e*a = a*e = a, \forall a \in G$.
iii. For every $a \in G$, there is $a^{-1} \in G$ such that $a*a^{-1} = a^{-1}*a = e$.
iv. $(a*b)*c = a*(b*c), \forall a,b,c \in G$.

If group theory is given directly and in formal form, students have difficulty understanding it. So, it was necessary to make students understand more easily. Inspired by Carter, in his book entitled *Visual Group Theory* [8], some activities were constructed, whereby the students would be expected to understand more about the group.

Strategy 1. Learning group by activities and technology. Different activities are arranged, so that by doing these activities, the student can understand different groups with different traits.

Activity 1. Exchange the position of two puppets. Students were given a set of four Indonesian puppets. The students were asked to make some steps, where for each step, they had to change the position of two puppets and observe the new order of the position of the puppets. With the help of a number representing each puppet, the students wrote down the results of the changes made. Through this activity, the students were expected to understand the concept of permutation groups and their properties more easily. The students were directed to realise that if two exchanges of positions are done in reverse order they will get different results. This leads to a noncommutative permutations group.

Figure 1: The puppets for the activity of permutation group concept.

Activity 2. Make a rotation of a rectangle. Students do a step-by-step rotation of a square with a rotation angle of 90°, 180°, 270° and 360°. Through this activity, the students are expected to more easily understand the concept of cyclic group and their properties.

Activity 3. Explore the characteristics of a group using *group exploration* software**.** After interpreting various groups through activities, students are introduced to various types of groups formally. Through software assistance, the students explore the properties of a group using group exploration software.

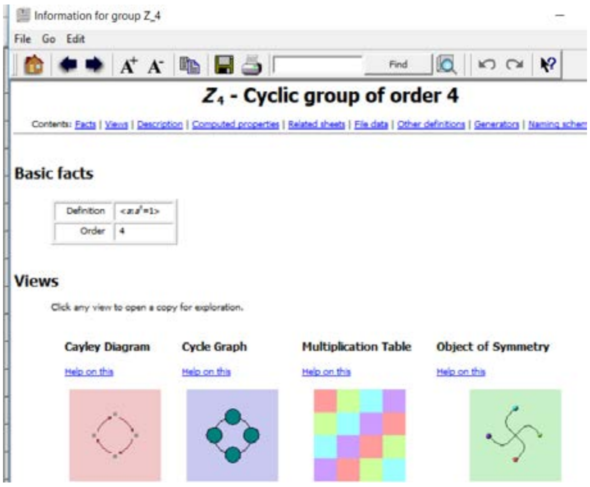


Figure 2: A partial view of group exploration of group module 4.

Strategy 2. Exploration of symmetry groups by batik and carving motifs.

The symmetry group of an object is the group of all transformations under which the object is invariant with composition function as the group operation. Example: the symmetry group of a rectangle has eight elements that are four rotations (with rotation angle 0°, 90°, 180°, 270°) and four reflections. Gallian, in his book, discussed the symmetry group of geometric objects [3]. Meanwhile, in this research, the students were asked to define all elements of a group formed by a given batik or carving motifs and determine the characteristics of the group. Students chose some batik and carving motifs from various regions to determine the symmetry group of the objects. The goal to be achieved from this activity was to make the student more familiar with batik motifs from various regions through the approach of its symmetry properties.

Figure 3: Some batik and carving objects of the symmetry group.

Strategy 3. Determine and find batik or carving motifs for each type of frieze group.

After studying the symmetry group of a finite object or bounded object (batik or carving motif), the object was changed by adding a strip (infinitely wide rectangle) with a repetition pattern in only one direction. The symmetry group of a strip is called a frieze group. So, the elements of a symmetry group of a strip are all transformations under which the strip is invariant with a composition function the same as the group operation. Because the strip has the repetitive nature of the pattern in one direction, then the translation transformation is always a member of the symmetry group of a strip. Example: the symmetry group of the following strip: a) has elements in form of translation only; and b) has translation and vertical reflection symmetry.

a)

b)

Figure 4: The pattern of a frieze group with a) translation only; and b) translation and vertical reflection.

There are five different types of symmetry for frieze patterns: 1) translation; 2) 2-fold rotation; 3) horizontal reflection; 4) vertical reflection; and 5) glide reflection. With this type of symmetry, a group frieze is classified into seven different patterns based on the combination of symmetry types that appear in the patterns. Mathematician Conway created names that relate to footsteps for each of the frieze groups: F1 hop (translation symmetry only); F2 step (translation and glide reflection symmetry); F3 sidle (translation and vertical reflection symmetry); F4 spinning hop (translation and rotation symmetry); F5 spinning sidle (translation, glide reflection and rotation symmetry); F6 jump (translation and horizontal reflection); and F7 spinning jump (translation, horizontal and vertical reflection, rotation).

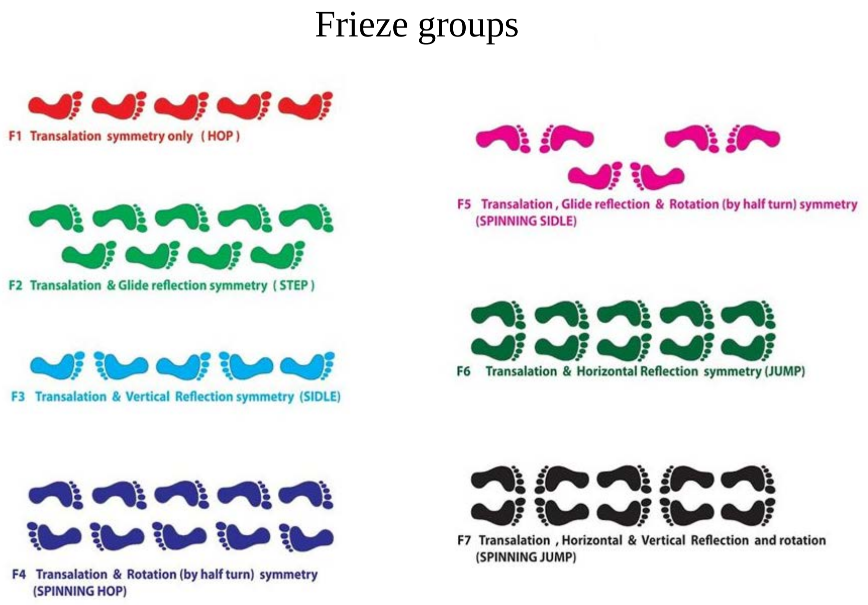


Figure 5: The seven frieze group patterns.

Students were given a variety of patterns and they had to determine what transformations mapped the pattern invariantly. This activity was directed, so that students could find out that there are only seven types of frieze group. After the students understood the concept of the frieze group, they determined the type of frieze group from some batik or carving images. They had to find pictures of batik or carvings motifs from different regions representing the seven types of frieze group and analyse the characteristics typical of the motif of some regions seen from the patterns.

Using Kali and Tess software or software that students chose, they were asked to design suitable patterns for each type of frieze group. Through this activity, the students are expected to be more familiar with local culture along with its characteristics and get to know the difference of batik motifs or carvings between regions in mathematical view (symmetrical pattern).

Strategy 4. Determine and find batik motifs for each type of wallpaper group.

If the symmetry group of a strip with repetition pattern goes in only one direction, then, for a wallpaper group the repetition pattern is in two directions (length and width) or can be described as an infinite perpendicular.

There are 17 different types of wallpaper or 17 possible plane symmetry groups. They are commonly represented using Hermann-Mauguin-like symbols or in orbifold notation [12].

Strategy used to study wallpaper group was similar to the frieze group.

RESULTS AND DISCUSSION

Although to understand groups through activities takes longer than conventional learning (direct material delivery), based on students' questionnaire responses, their results demonstrate a better understanding of group concepts than without such activities. By completing the tasks associated with the culture, students are also more familiar with the local culture and they feel the benefits of mathematics when they learn to see the characteristics and the differences of batik or carving motifs the area of Indonesia. As well as using various software, students had literature about the technology and this helped them to create the desired motives, and made it easier for them to understand the concept of the group and its properties. Learning with this strategy gets a positive response from students. Some students said that before they were not too familiar with batik motifs, but by doing the task, they realised that the motif of batik from one area is different from other areas, especially when viewed from the nature of symmetry.

Some of the results achieved by students are shown in Table 1.

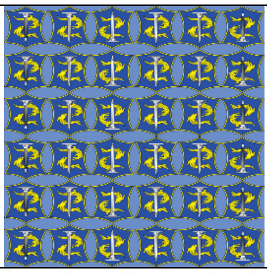
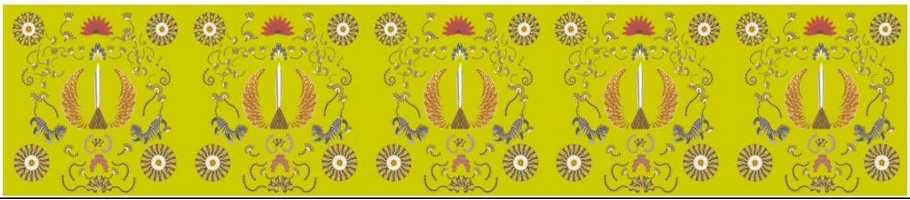
Batik from Bali type F1 (translation only)	
Batik from Java type F2 (translation and glide reflection)	
Batik from Papua type F3 (translation and vertical reflection)	
Mathematical-themed designs by students with Kali program	
Surabaya icon design by students with jBatik software (frieze group type F1)	
Surabaya icon batik design by students (wallpaper group)	
Surabaya icon batik design by students with jBatik software	

Table 1: Some works of students.

CONCLUSIONS

Learning group concepts through activities has enabled students to understand the concepts and their properties better, and give a concrete meaning to abstract mathematical statements.

Strategies used by incorporating batik motifs or engraving as objects in group theory learning make students more familiar with local culture, and they learn to appreciate it.

By using software in learning, students can understand more easily the concept and the properties of groups and through Kali and Tess programs, students can be creative in making various motifs in accordance with the patterns they studied in symmetry group activity. This activity increases students' motivation in learning group theory as they feel the application of the theories learned in life.

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